**Program 8**

\documentclass{article}

\usepackage{amsthm}

% Define theorem-like environments

\newtheorem{theorem}{Theorem}

\newtheorem{definition}{Definition}

\newtheorem{corollary}{Corollary}

\newtheorem{lemma}{Lemma}

\begin{document}

\title{Numbered Theorems, Definitions, Corollaries, and Lemmas in the Document}

\date{}

\maketitle

\begin{theorem}

(Pythagorean Theorem) In a right-angled triangle, the square of the length of the hypotenuse

is equal to the sum of the squares of the lengths of the other two sides.

\begin{equation}

a^2 + b^2 = c^2

\end{equation}

\end{theorem}

% Presenting a definition with an example

\begin{definition}

(Prime Number) A prime number is a natural number greater than 1 that is not divisible by

any number other than 1 and itself.

\begin{itemize}

\item Example: 2, 3, 5, and 7 are prime numbers.

\end{itemize}

\end{definition}

% Presenting a corollary with an example

\begin{corollary}

(Euclid's Corollary) There are infinitely many prime numbers.

\begin{itemize}

\item Proof: Assume there are finitely many primes. Let them be $p\_1, p\_2, \ldots, p\_n$.

Consider the number $N = p\_1 \cdot p\_2 \cdots p\_n + 1$.

This number is not divisible by any of the primes $p\_1$ through $p\_n$.

Therefore, there must be a prime factor not in the list, contradicting the assumption that there

are only finitely many primes.

\end{itemize}

\end{corollary}

% Presenting a lemma with an example

\begin{lemma}

(Basic Arithmetic Identity) For any real numbers $a$ and $b$, we have:

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\begin{equation}

(a + b)^2 = a^2 + 2ab + b^2.

\end{equation}

\end{lemma}

\end{document}